

XXXVII. *A Treatise on Rivers and Canals.* By Theod. Aug. Mann, *Member of the Imperial and Royal Academy of Sciences at Bruffels; communicated by Joseph Banks, Esq. P. R. S.*

Read June 24, 1779.

TO JOSEPH BANKS, ESQ. P. R. S.

S I R,

YOUR election to the Presidency of the first literary and scientific Society in the world, to a chair so long and so gloriously occupied by the great NEWTON; joined to the friendship you have been pleased to honour me with since my being first known to you: has encouraged me to send you something of my composition, as the best way of expressing my sincere respect and attachment to you, and my profound veneration for the illustrious Body which has chosen you for its head. Though various circumstances, by carrying me very early into foreign countries, have made me from my youth almost
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an alien to my native soil, and put me in a situation which apparently must make me ever remain so; yet, neither time nor distance could ever weaken, much less obliterate, my tender attachment to it, or my ardent wishes for its welfare.

These considerations will, I hope, merit a favourable acceptance from the Royal Society of the following piece, which I have the honour of addressing to you; and an indulgent condescension for its imperfection in every respect, and particularly in point of style. Five and twenty years absence from my native country, and the necessity of conversing during that time in different foreign languages, must unavoidably have filled mine, without my being sensible of it, with idioms and expressions in no wise English.

As to the subject I have undertaken to treat on this occasion, I was guided in the choice thereof by the motive of saying something that might be useful to my native country. The great number of extensive and magnificent canals, which have been cut through almost every part of England of late years, for the use of internal navigation, and which do honour to the public spirit of the nation, merit to be considered in a scientific as well as in a commercial light. Their waters have their laws of motion different in many cases from those of

rivers: they are liable to many accidents which the others are not, and of a different nature. These accidents do not become sensible till many years after their construction, and are better prevented in time than remedied when they happen. I have long lived in a country famous for its navigable canals, and have been much employed, under the eyes of the government of it, upon that subject. I only mention this to shew, that I have not undertaken to treat a subject to which I am an utter stranger.

There are, moreover, many considerations concerning the laws of motion in rivers and canals in general, the velocity of their currents in proportion to the quantity of their declivity, and the means of ascertaining therefrom the respective heights of the interior parts of continents, which merit the attention of a natural philosopher. I shall venture to offer my thoughts and observations (some of which, I believe, are new) on all these subjects in the ensuing Dissertation, which I submit entirely to the judgment of the Royal Society, and shall esteem myself happy if I succeed in it, so as to be of any use to my country, and to be able to testify, at the same time, my profound respect and veneration to you, DEAR SIR, and to the illustrious body over which you preside.

SECTION I.

Different uses for which canals are made, with an account of the principal authors who have wrote concerning them.

1. Artificial canals are to be considered in a double light; as facilitating commerce by means of internal navigation, and as preventing inundations by carrying off the too great abundance of water from low and flat countries, such as are Holland, Flanders, &c. In these last named countries they serve at once for both purposes; and it is in this double light that I shall consider them in the ensuing discourse. If canals for draining have sluices upon them, particularly at the end whereby they discharge their waters, as is universally the case in the Low Countries, they differ in no wise from navigable canals: if they have nothing to sustain their waters in them, they are to be considered in every respect as rivers or rivulets, and follow the same laws. It must, therefore, be carefully kept in mind, that whenever I mention canals, I mean those only whose waters are kept up by sluices, and never those without them, which I include, without distinction, under the common appellation of rivers; for they

they are no more than artificial ones. If I mistake not, all the navigable canals in England are of the first sort; that is, have their waters kept up, and let off by sluices. This necessary distinction will take away all ambiguity from what I have to say on canals throughout the following discourse.

2. But that I may fulfil the task I have undertaken, it is necessary first of all to lay down such principles on the nature of rivers and canals in general as have been demonstrated true both by calculation and experience; to the end, that we may deduce from thence the true laws of motion of their waters, and the quantity of declivity of their beds: for this purpose, and because a large volume would hardly suffice to comprise all the demonstrations of these principles, which, consequently, I am obliged to omit in this treatise, it will not be amiss to mention the principal authors who have treated this subject in different ages and countries, in whose works the demonstrations of all the principles I shall lay down may be found, if any one doubts the truth of them. These are the following.

SEXTUS JULIUS FRONTINUS, de Aquæ-ductibus Urbis Romæ, cum Notis POLENI, impress. 1722.

JOHN BAPTIST ALEOTTI, Hydrometrician to the Duke of Ferrara, and to Pope CLEMENT the VIIIth.

* DON BENEDICT CASTELLI, Benedictine Abbot, de
Menfurâ Aquarum Currentium.

J. B. BARATTERI, de Architettura d'Acque, lib. VI.
Piacenza, in folio, 1656.

ALEXANDER BELTINZOLI, of Cremona.

NICOLAUS CABEUS, in Libris Meteorum.

GALILEI GALILEO.

* JOH. BAPT. BALIANI, de Motu Liquidorum.

* JOH. BAPT. RICCIOLI, Geographiæ et Hydrographiæ
Reform. libro VI. cc. 29. et 30.

* CLAUDE MILLET DESCHALES, de Fontibus et Flu-
minibus, à prop. 39. usque ad 56.

VARENNIUS, General Geography, with Dr. JURIN's
and Dr. SHAW's Notes, edit. of 1765, vol. I. from page
295 to page 358.

Dr. JURIN, in the Philosophical Transactions, N^o 355.
page 748. et seq.

MARIOTTE, Traité du Mouvement des Eaux.

VARIGNON, Memoires de l'Academie des Sciences de
Paris, pour 1699 et 1703.

* Sir ISAAC NEWTON, Princip. Mathem. lib. II. § 7.
page 318 et seq. edit. 1726.

* DANIELIS BERNOULLI, Hydrodynamica, in quarto,
Argentorati, 1738.

* DOMENICHE GUGLIELMINI, della Natura de Fiumi, Bononiæ, 1697, in quarto. Ejusdem de Mensurâ Aquarum fluentium, Bononiæ, in quarto.

* JOH. POLENUS, de Castellis et de Motu Aquæ mixto, Patavii, 1697, 1718, 1723.

* RACCOLTA d'Autori che trattano del Moto dell' Acque, Fiorenza, 1723, 3 vol. quarto, cum fig.

JAC. HERMANNUS, in Phoronomia, cap 10. page 226. et seq.

CHRIST. WOLF, Curf. Mathem. Hydraulicæ, cap. VI. edit. Genevæ, in quarto, 1740.

M. DE BUFFON, sur les Fleuves, dans son Histoire Naturelle, tom. H. p. 38—100. de la 1^{re} edit. en 12mo.

Several Memoirs upon this Subject in the Collection of the Royal Academy of Sciences of Paris, particularly those of M. PITOT, in the volumes for 1730 and 1732.

S'ERAVESANDE, in Elementis Physicæ, tom. I. lib. II. cap. 10.

* R. P. LECCHI S. J. Hydrostatica, Mediolani, 1765. In this excellent work are several pieces by Father BOSCOVICH upon the same subject.

* STATTLERI Physica, vol. III. p. 232—286. de cursu Fluminum, ejusque Mensuratione et Directione. Aug. Vindel. 1772, 8 vol. in octavo. This author gives many late observations and experiments on the motion and

measure

measure of currents, as those of ZENDRINI, HIMENII, &c.

Two other authors have lately wrote upon rivers and canals, but their works are not yet come to my hands; to wit,

Father FRISI, an Italian Barnabite, Professor of Mathematics at Milan.

M. DE LA LANDE, of the Royal Academy of Sciences at Paris, who has just published a History in folio, with plates, of all the Navigable Canals in the World that have come to his Knowledge.

Among the above authors, those marked with an asterisk (*) are they who have treated the subject in question with the greatest exactness or most extent; and it is from them chiefly that I shall lay down such principles and laws of action in rivers and canals as regard the subject I have taken in hand. By this means I shall avoid advancing any thing upon so important a matter, but what is founded upon the most certain and exact experiments, and conformable to what are demonstrated to be the real and unalterable laws of nature.

S E C T I O N II.

The theory of rivers and canals.

I. DEFINITIONS.

3. A river is a greater or lesser quantity of water which runs constantly, by its own gravity, from the more elevated parts of the earth, towards those which are more depressed, in a natural bed or channel open above.

4. If this bed or channel is artificial, and has been dug by hands, it is called *a canal*, of which there are two kinds; those where the channel is every where open, and without sluices, which I call an *artificial river*; and those where the waters are kept up or let off by the means of sluices: it is this second sort which I shall call hence-forwards by the proper name of *a canal*.

5. *A river is said to persevere in the same state* so long as there runs off an equal quantity of water in the same time, without any increase or diminution, so that it remains always at the same height in the same place. When the circumstances are different from this, it is said respectively that *a river increases or diminishes*.

6. *A section* of the bed of a river or canal is a plane drawn perpendicular to the bottom of the bed and to the direction

direction of the stream of water, and whose limits are those of the water itself which runs off in that place.

7. I call *sections of equal velocity*, all those where the water runs with equal velocity; and *sections of greater or lesser velocity*, those where the water runs faster or slower respectively, and when compared to others.

8. I call *mean velocity of a current or stream of water*, that which a river or canal would have, if all the parts thereof, to wit, those of the bottom, the sides, the middle, and the surface of the same section, ran with an equal velocity, in such a manner that there would pass just as much water in the same time by this uniform motion, as there does now actually pass by the irregular flowing of the stream.

II. PROPOSITIONS, or laws of action in rivers and canals.

9. The motion of water in rivers proceeds from the same principle which produces the descent of heavy bodies upon inclined planes:

I I. The descent of heavy bodies upon inclined planes follows exactly the same laws as those observed in the descent of heavy bodies in a perpendicular line towards the center of the earth; that is,

1st, They descend by a motion uniformly accelerated.

2dly, The

2dly, The spaces, run over by heavy bodies which fall perpendicularly by a motion uniformly accelerated, are in *a duplicate ratio of the times and velocities* respectively.

3dly, These spaces, in equal times, augment in the same ratio as the odd numbers in progression 1, 3, 5, 7, 9, 11, &c.

4thly, Therefore, both the times and the velocities are in a *sub-duplicate ratio of the spaces run over*.

5thly, It is demonstrated by the principles of mechanics, that the velocity acquired by a heavy body descending freely upon an inclined plane, in a given time, is to the velocity which the body would acquire in the same time, by falling perpendicularly, as the height of the inclined plane is to its length.

6thly, From whence it follows, that the velocities which bodies acquire in their descent upon inclined planes, are in *a direct ratio of the square roots of the quantity of inclination or declivity of the planes*.

11. So that when water flows freely upon an inclined bed, it acquires a velocity, which is always as the square root of the quantity of declivity of the bed.

12. In an horizontal bed, opened by sluices or otherwise, at one or both ends, the water flows out by its gravity alone; and the flowing is quicker or slower in a

rect ratio of the respective heights of the water, by reason of the weight of the superior waters upon the inferior.

13. From hence (N^o 11, 12.) it follows, first, that as much as the declivity of the bed or channel of a river is greater, so much also will the velocity of the flowing waters be proportionably increased.

2dly, As much as the water in an horizontal bed is deeper, so much will the velocity of the current be increased; and this velocity will diminish in proportion to the decreasing depths of the water in the bed.

3dly, Abstracting from the resistance caused by the bottom and sides of the bed, as much nearer as the water is to the bottom, so much will its motion be accelerated; not only because the inferior waters are more compressed by the superior in proportion to their greater depth; but also because the inferior ones have a greater declivity than the superior, by reason of their greater depth in the bed, where they are more depressed with respect to the elevation of their common source or spring. But these different velocities of the upper and lower waters in the same section of the bed (abstracting from the friction of the bottom and sides) approximate indefinitely to each other in proportion to the length of the channel, but still without a possibility of their ever becoming equal in fact, if they met with no resistance from the bed.

14. There-

14. Therefore, the motion of water flowing freely in an inclined channel, is accelerated by its own weight combined with the quantity of declivity in the bed.

Nevertheless, the velocity of waters which flow in an inclined bed, during their actual flowing, is not accelerated by the weight which the inferior waters sustain from the superior ones, in case the lower parts have already, by the declivity of the bed, a greater velocity than that which the weight of the superior ones impresses upon them. The reason of which is, that no body which follows another with a lesser velocity can act by impulsion upon that which precedes it with a greater velocity, as is the case with regard to these superior and inferior waters. But the weight of the upper waters begins to accelerate the lower as soon as they fall into an horizontal bed, or one that is so nearly horizontal as to destroy the greater velocity of the lower waters above that of the upper.

15. The velocity of rivers depends sometimes upon the sole declivity of their beds; sometimes also upon the sole gravity of their waters: and if these two causes sometimes act together, the effect produced is only the respective excess of the one above the other. It often happens in the same section of a river, that the acceleration of velocity in the inferior parts proceeds from the weight of

the upper waters, while that in the upper parts proceeds from the declivity of the bed.

From whence it follows, that in rivers which have little declivity, it is the depth of the waters which contributes most to accelerate their current; and in those whose beds have most declivity, it is the descent of gravity upon an inclined plane which has the greatest share in producing this acceleration.

To find whether the water in a part of a river where the bed is nearly horizontal flows by the velocity acquired in the preceding declivities, or by the compression of the upper waters upon the lower in that place; a pole must be thrust down to the bottom, and held perpendicular to the current of the water, with its upper end above the surface: if the water swells and rises immediately against the pole, it shews that its flowing is by virtue of a preceding declivity: if, on the contrary, the water stops for some moments before it begins to rise against the pole, it is a proof that it flows by means of the compression of the upper waters upon the lower.

16. The *absolute height or elevation* of the surface of a river which perseveres in the same state (N^o 5.) continually decreases, as the distance in the river from its source increases; by reason that its bed must continually incline and tend towards the center of the earth.

17. The

17. The velocity of each particle of water in a regular channel, that is, where the bed is a regularly inclined plane, may be determined by drawing a perpendicular from the particle proposed to the horizontal curve which passes through the spring, or that point of the river where the particle in question begins to acquire its velocity. For the velocity which this particle would acquire, in falling freely along the said perpendicular, is the same as that which it has acquired in its descent along the inclined plane of its bed.

18. So long as *a river perseveres in the same state* (N^o 5.) there flows an equal quantity of water in equal times, how unequal soever the sections be through which they flow; and, consequently, where the section of the river is greater, the velocity of the flowing water is less; and where the section is less, the velocity is greater; always in an inverse proportion. From hence may be deduced the following and other similar propositions.

1st, Through equal sections, in equal times, and with equal velocity, there must flow equal quantities of water.

2^{dly}, Through equal sections, in equal times, but with unequal velocities, the quantities of water which flow, are in a direct ratio of the respective velocities.

3^{dly}, Through

3dly, Through unequal sections, in equal times, and with equal velocities, the quantities of water which pass are in a direct ratio of their respective sections.

4thly, Through unequal sections, and with unequal velocities, the quantities of water which flow in equal times, are in a combined ratio of the sections and mean velocities (N^o 8.) together.

In a word, the sections of the bed, the mean velocities, the times of flowing, and the quantity which flows, are universally in a combined ratio together; and this combination is what is called *the momentum of a river*; and this *momentum of the same flowing water is universally equal*.

19. From hence may easily be deduced the principles for calculating the quantity of diminution of the water in a lake, pond, or vessel, by any determinate flowings whatsoever: for as the surface of the lake, &c. is to the section of the current which carries off the waters; so is the mean velocity of the current in this section to the decrease of the waters in the lake, &c. and *vice versa*.

III. *On the nature of rivers and flowing waters.*

20. Rivers contain divers inherent causes of the acceleration of their motion.

1st, Their springs are either in mountains or on high grounds, and it is by the descent of the waters from these elevations that they acquire a velocity and acceleration of motion sufficient to sustain and propagate it through the rest of their course.

2dly, The cohesion of the particles of a fluid, *in a bed ever so little inclined,* is a second cause of acceleration of motion in the fluid; because, by their mutual attraction, those particles which begin first to flow draw after them those which are contiguous, these the following, and so on *ad infinitum.*

3dly, Moreover, where a river, by flowing in a bed nearly horizontal, has lost a great part of the velocity which it had acquired in the preceding declivities, and the bed by this means is become large and shallow, which consequently again augments the slowness of the current; it may, however, recover a part of its velocity, even in the same horizontal bed, by augmenting the depth thereof, and diminishing its breadth; for by this means the weight of the superior waters upon the inferior

rior is increased, and consequently the velocity of the whole is augmented (N^o 13. 18.). In the same manner a junction of rivers in the same bed, by excavating and deepening it, augment the velocity of the common current, as we shall shew more particularly hereafter.

21. On the other hand, flowing waters meet with many powerful causes of resistance to their motion, which tend continually to diminish their velocity. Such are the following:

1st, The attraction and continual friction of the bottom and the sides of the bed, contribute greatly towards retarding the motion of the water.

2dly, The same effect is produced likewise by the many obstacles which they meet with in their way; such as inequalities in the bottom and sides of the channel, banks of sand and mud, rocks, trunks of trees, and other such things.

3dly, The many windings and angles made in their course, which produce so much the more resistance and hindrance to the motion of the water, as the course varies more and oftener from a right-line.

4thly, The diminution of their declivity the farther they recede from their springs; this being generally the least towards their mouths, which are for the most part in extensive plains.

Finally,

Finally, the natural cohesion of the particles of water *in an horizontal bed* contributes to retard its motion precisely by the same force which contributes to accelerate it *in an inclined bed*. By diminishing or taking away the above obstacles to the free motion of water in rivers and canals, the velocity of their currents will be increased in the same proportion, and thereby also all the dangers and ravages of inundations may be prevented, as we shall shew hereafter.

But if some or all of these causes, in a greater or lesser degree, did not exist in rivers of considerable depth and declivity, it is demonstrated ^(a), that the velocity of their currents would be accelerated to twelve, fifteen, and, in some cases, even to twenty times more than it is at present in the same rivers, whereby they would become absolutely unnavigable.

22. The waters in a river or open canal have their motion accelerated, so long as the effects proceeding from gravitation, declivity, depth, in a word, so long as *the sum of accelerations surpasses the sum of resistances*.

When these different sums become equal to each other, the motion of the water is neither accelerated nor retarded, but remains equal, till something anew destroys the *equilibrium*.

(a) Vide LECCHI, Hydrostat. et STATTLER, Physic. tom. III. p. 252.

When the sum of resistances and causes of retardation is greater than the sum of accelerating causes, the velocity of the river is diminished in proportion to the excess.

23. The *percussion* of the waters of a river against an obstacle which is opposed to their motion, is the action of the waters striking against that obstacle; and the principles for calculating the quantity of this percussion, or the effects which any obstacles whatever produce in the motion of rivers, by known forces, and in determinate times, are as follows:

1st, The percussion of the water of a river against any obstacle whatever is universally in a compound ratio of the quantity of the plane or planes which the obstacle opposes to the current of water, of the sine of the angle of incidence which the direction of the current makes with these planes, and of the square of the velocity of the said current.

2dly, The resistance, therefore, which the bed of a river opposes to its current from any particular obstacles in it, is in a compound ratio of the magnitude and situation of the planes of these obstacles, together with the square of the current's velocity in the place where those obstacles are found.

3dly, The accelerating force of a river, or that by which it surmounts the resistance of its bed in any one

place, compared to that in another, is in a compound ratio of the mass of water and of the velocity of the current in those places respectively.

24. In different parts of the same river, the velocity of the current is greater in a direct proportion of the greater declivity of the bed; because the relative gravity of the flowing particles augments in that ratio.

25. But in the same section of a river, the superior parts, and those which are farthest from the bottom and the sides, will continue their course by the sole cause of the declivity of their bed, *how little soever it be*; because these waters not being retarded by the friction of the bottom and sides of the bed, or hardly by any other obstacle whatever, *the least possible deviation from a level will produce a current*. But the waters at the bottom of a river, both because of their friction against it, and of the irregularities which are almost every where found in it, will lose that little motion which a very small declivity can give them, and their motion in that case will be produced alone by the compression of the superior waters upon them.

The inferior waters which thus acquire their motion from the weight of the superior ones upon them, communicate reciprocally a part of their motion, by means of the natural cohesion of the particles together (N^o 20.)

to the superior ones, which in an horizontal bed, without this cause, would have no other motion than that which is impressed upon them by the impulsive force of the waters descending from their elevated springs.

From whence it appears, that the superior and inferior waters in a river communicate reciprocally a part of their motion to each other; but this can never go beyond a certain point or *maximum*, which is always proportionable to the *momentum* of the river in that place (N^o 18.).

It follows from hence, that the greatest velocity of a river, running in a right-line, is in *the center of its section* (N^o 6.); that is to say, in that point which is the farthest possible from the surface of the water and from the bottom and sides of the bed, all taken together. This part has the advantage of one half of the depth of water pressing upon it, and it is exempt from the friction of the bottom and sides of the bed which are there overcome and vanish by the perpendicular compression.

On the contrary, the least velocity of the water is at the bottom and sides of the bed, because it is there that the resistance produced by friction is greatest, from whence it is communicated to the other parts of the section in an *inverse duplicate proportion of the distances from the bottom and sides combined together*, until it becomes a
negative

negative quantity, where the effect vanishes, or is reduced to nothing.

26. The best and most simple method of measuring the velocity of the current of a river or open canal; that I know of, is the following:

Take a cylindrical piece of dry, light wood, and of a length something less than the depth of the water in the river: round one end of it let there be suspended as many small weights as may be necessary to keep up the cylinder in a perpendicular situation in the water, and in such a manner that the other end of it may just appear above the surface of the water. Fix to the center of that end which appears above water a small and straight rod, precisely in the direction of the cylinder's axis; to the end, that when the instrument is suspended in the water, the deviations of the rod from a perpendicularity to the surface of it may indicate which end of the cylinder advances the fastest, whereby may be discovered the different velocities of the water at different depths; for if the rod inclines forwards according to the direction of the current, it is a proof that the surface of the water has the greatest velocity; but if it inclines back, it shews that the swiftest current is at the bottom; if it remains perpendicular, it is a sign that the velocities at the surface and bottom are equal.

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This instrument being placed in the current of a river or canal receives all the percussions of the water throughout the whole depth, and will have an equal velocity with that of the whole current *from the surface to the bottom* at the place where it is put in, and by that means may be found, both with ease and exactness, the mean velocity of that part of the river for any determinate distance and time.

But to obtain the mean velocity of the whole section of the river, the instrument must be put successively both in the middle and towards the sides, because the velocities at those places are often very different from each other. Having by this means found the *difference of time required for the currents to run over an equal space*; or, *the different distances run over in equal times, the mean proportional* of all these trials, which is found by dividing the common sum of them all by the number of trials, *will be the mean velocity of the river or canal.*

If it be required to find the velocity of the current only at the surface, or at the middle, or at the bottom, a sphere of wood, of such a weight as will remain suspended in equilibrium with the water at the surface or depth which we want to measure, will be better for the purpose than a cylinder, because it is only affected by the
water

water of that sole part of the current where it remains suspended.

It is very easy to guide both the cylinder and the globe in that part which we want to measure, by means of two threads or small cords, which two persons must hold and direct, one on each side the river; taking care at the same time neither to retard nor accelerate the motion of the instrument.

Several other methods have been invented for determining the velocity of the currents of rivers and canals, which may be seen in most of the authors enumerated in the beginning of this essay (N^o 2.)

IV. Application of the preceding laws of the acceleration and retardation of currents to rivers and canals in general, from whence are deduced the various means of preventing or remedying the defects and inconveniencies which must necessarily happen to them in a series of years.

27. By combining together all we have said hitherto upon the nature and theory of motion in rivers, and particularly in the articles 13. 18. 20. 21. and 23. it follows evidently, that *the deeper the waters are in their bed in proportion to its breadth, the more their motion is accelerated;*

celerated; so that their velocity increases in an inverse ratio of the breadth of the bed, and also of the greatness of the section; from whence are deduced the two following universal practical rules:

1st, To augment the velocity of water in a river or canal, without augmenting the declivity of the bed, we must *increase the depth and diminish the breadth of its bed.*

2dly, But to diminish the velocity of water in a river or canal, we must, on the contrary, *increase the breadth and diminish the depth of its bed.*

The above proposition is perfectly conformable to observation and experience; for it is constantly seen, that the current is the swiftest where the waters are deepest and the breadth of the bed the least; and that they flow slowest where their depth is the least and the breadth of the bed the greatest. "The velocity of waters," says M. DE BUFFON ^(b), "augments in the same proportion as the section of the channel through which they pass diminishes, *the force of impulsion from the back-waters being supposed always the same.* Nothing," continues he, "produces so great a diminution in the swiftness of a current as its growing shallow; and, on the contrary, the increase of the volume of water augments its

(b) Hist. Nat. tom. II. p. 53. 60. edit. in 12mo.

“ velocity more than any other cause whatever. The celebrated WOLF, in his *Hydraulics* ^(c), assures us, that “ it is a constant and universal practice, for accelerating “ the current of waters, to deepen the bed, and at the “ same time to render it narrower.”

28. When the velocity which a river has acquired by the elevation of its springs and the impulse of the back-water, is at last totally destroyed by the different causes of resistance which we have enumerated above (N^o 21.) becoming equal or greater than the first, the bed and current at the same time being exactly horizontal, nothing else remains to propagate the motion, except *the sole perpendicular compression of the upper waters upon the lower, which is always in a direct ratio of their depth.* But this necessary resource, this remaining cause of motion in rivers, augments in proportion as all the other diminish, and as the want of it increases: for as the waters of rivers in extensive plains lose the acceleration of motion acquired in their descent from their springs, their quantity accumulates in the same bed by the junction of several streams together, and their depth increases in consequence thereof. This junction and successive accumulation of many streams in the same bed, which we see universally in a greater or less degree in all rivers throughout the known world, and which is so absolutely

(c) N. 224.

necessary to the motion of their waters, can only be attributed, says Signor GUGLIELMINI ^(d), to the infinite wisdom of the supreme Author of Nature.

29. The velocity of flowing waters is very far from being in proportion to the quantity of declivity in their bed: if it was, a river whose declivity is uniform and double to that of another, ought only to run with double the swiftness when compared to it; but in effect it is found to have a much greater, and its rapidity, instead of being only double, will be triple, quadruple, and sometimes even more: for its velocity depends much more on the quantity and depth of the water, and on the compression of the upper waters on the lower, than on the declivity of the bed. Consequently, whenever the bed of a river or canal is to be dug, the declivity must not be distributed equally throughout the whole length; but, to give a swifter current to the water, the declivity must be made much greater in the beginning of its course than towards the end where it disembogues itself, and where the declivity must be almost insensible, as we see is the case in all natural rivers; for when they approach near the sea their declivity is little or nothing, yet they flow with a rapidity which is so much greater, as they contain a greater volume of water: so that in

(d) Della Natura de Fiumi.

great rivers, although a large extent of their bed next the sea should be absolutely horizontal, and without any declivity at all, yet their waters do not cease to flow, and to flow even with great rapidity, both from the impulsion of the back waters, and from the compression of the upper waters upon the lower in the same section.

30. Whoever is well acquainted with the principles of the higher geometry, will easily perceive that it would be no difficult matter to dig the bed of a canal or river, that *the velocity of the current should be every where equal*. It would be only giving it the form of a curve along which a moving body should recede from a given point, and *describe spaces every where proportional to the times*, allowance being made therein for the quantity of effect of the compression of the upper waters upon the lower. This curve is what is called the *Horizontal Isochronic*, being the flattest of an infinity of others which would equally answer the problem where fluids were not concerned. Upon these curves may be seen LEIBNITZ, HUYGHENS, and the two BERNOULLI'S, who were the first that determined and analysed them, and also many succeeding geometricians, if any one is desirous to occupy himself in such speculations as are more curious than useful, which is not my purpose in this treatise.

31. Notwithstanding all we have said concerning the necessity of augmenting the depth of a river in a greater proportion than its breadth, if we would accelerate its current; yet it is certain, that this can only be done to a certain point, without destroying that equilibrium which ought to reign between the depth and the breadth of the section of the stream, and thereby putting the river into a state of continual violence, which will incessantly exert itself to the destruction of the banks and wiers made to keep it in, and that action will always exert itself in a direct ratio of the greater or less want of equilibrium, as it would be easy to demonstrate by the principles of hydraulics. These same principles give likewise the just proportions of this equilibrium between the perpendicular and lateral compression of the water in any river or canal whatsoever, which vary in an inverse proportion, according to the different degrees of the declivity and velocity of the current; and in a direct one of the greater or less coherence and hardness of the substances which compose the bed. Rivers which flow in beds composed of homogeneous matter of little consistency, such as sand, &c. are always more broad than deep, when compared to those which run in beds of matter of greater tenacity. It is manifest, that the equilibrium here spoken of is real,

because rivers remaining in the same state only widen their beds to a certain pitch which they do not surpass.

32: M. DE BUFFON remarks, " That people accustomed to rivers can easily foretell when there is going to be a sudden increase of water in the bed from floods produced by sudden falls of rain in the higher countries through which the rivers pass. This they perceive by a particular motion in the water, which they express in their dialect, by saying that *the river's bottom moves*; that is, the water at the bottom of a channel runs off faster than usual; and this increase of motion at the bottom of the river always announces a sudden increase of water coming down the stream. Nor does their opinion therein," continues the same author, " seem to be ill-grounded on the nature of things; for the motion and weight of the waters coming down, though not yet arrived, must act upon the waters in the lower parts of the river, and communicate by impulsion part of their motion thereto; since a canal or river contained in its bed is to be considered in some degree as a column of water contained in a long tube, where the motion is communicated at once throughout the whole length." In a river or canal, open above, it is only communicated to a certain distance; that is, as far as the impulsive force of the new increase and superior rapidity

rapidity of the back-waters acts upon the stream, which will always be as far as till this force is gradually, and at last wholly, destroyed by the superior gravitation of the super-incumbent waters in the stream. Something of the same kind happens when a very great additional weight comes suddenly upon the surface of a river or canal; for instance, by the launching of a ship or of several boats together upon it. These causes increase the velocity of the water in the lower parts of the bed, and moreover retard its motion at the surface, which effect may properly be called *making the river's bottom move*. For the same reason, the increase of weight of the waters in a sudden flood, as well as the increase of their impulsive force, must contribute to produce this effect, and, by increasing the motion in the bottom of the river, may hinder, for some space of time, the stream from sensibly rising in the bed.

33. All obstacles whatever in the bed of a river or canal, such as rocks, trunks of trees, banks of sand and mud, &c. must necessarily hinder proportionably the free running off of the water; for it is evident, from what we have said, that the waters so far back from these obstacles, until the horizontal level of the bottom of the bed becomes higher than the top of the obstacles, must be intirely kept up and hindered from running off in proportion

portion thereto (N^o 23.). Now as the waters must continue to come down from their sources, if their free running off is hindered by any obstacles whatever, their relative height back from them must necessarily be increased until their elevation, combined with the velocity of their current proceeding from it, be arrived to such a pitch at the point where the obstacles exist, as to counterbalance the quantity of opposition or impediment proceeding from thence, which frequently does not happen until all the lower parts of the country round about are laid under water.

34. Now it is certain from all experience, that the beds of rivers and canals in general are subject to some or others of the obstacles above mentioned. If rocks or trees do not bar their channels, at least the quantity of sand, earth, and mud, which their streams never fail to bring down, particularly in floods, and which are unequally deposited according to the various windings and degrees of swiftness in the current, must unavoidably, in course of time, fill up, in part, different places in the channel, and thereby hinder the free running off of the back waters. This is certainly the case, more or less, in all rivers, and in all canals of long standing, as is notorious to all those well acquainted with them. Hence, if these accidents are not carefully, and with a constant

constant attention prevented, come inundations, which sometimes lay waste whole districts, and ruin the finest tracts of ground, by covering them with sand: hence rivers become unnavigable, and canals uselefs, for the purposes for which they were constructed. Canals, in particular, by reason that their waters for the most part remain stagnant in them, are still more liable than rivers to have their beds fill up by the subsiding of mud, and that especially for some distance above each of their sluices; inasmuch, that if continual care is not taken to prevent it, or remedy it as often as it happens, they will soon become incapable of receiving and passing the same vessels as formerly. Nay, the very sluices themselves, if the floors of their bottoms are not of a depth conformable to the bed of the canal, will produce the same accidents as those we have been speaking of; for if they are placed too low, they will be continually filling up with sand or mud; if too high, they have the same effect as banks or bars in the bed of a river, that is, they hinder all the back-waters under their level from running off, and soon fill up the bed to that height by the subsiding of mud. This effect is much accelerated by the shutting of the lower sluices, which makes a great volume of water reflow back to those next above them, till the whole is filled and becomes stagnant. Now it is evident, that

that this state of things must contribute far more to the subsidency of mud and all other matters brought down by the waters in canals, than can be the case in rivers whose currents constantly flow.

35. I do not suppose that these inconveniencies can have yet manifested themselves by any very sensible effects in the many new canals and sluices lately constructed in England; but as the same causes do not cease to act more or less every where, the effects which necessarily follow from them will likewise become more and more sensible, unless continual care be taken to prevent them. The waters of all rivers and canals are from time to time muddy: their streams, particularly during rains and floods, carry along with them earth and other substances which subside in those places where their currents are the least, whereby their beds are continually raised: so that the successive increase of inundations in rivers, and of unfitness for navigation in canals, when they are neglected and left to themselves, is a natural and necessary consequence of the state of things, which no intelligent person can be at a loss to account for; and yet I have known whole countries remain in this habitual state of negligence to their very great detriment.

36. Having thus shewn the principal accidents which rivers and canals are liable to, with the causes of them, I

I shall proceed to point out the most efficacious methods of preventing them, or at least of diminishing their effects. Perhaps it would have been more proper to have deferred doing this till I should have said all I have to say upon the nature of rivers and canals: however, I shall forego the more scientific order of things, for the sake of bringing the means of remedying the accidents and inconveniences which happen, nearer to the causes that produce them, whereby their connexion and efficacy may be better judged of. For this end, I shall here lay down, briefly and in general terms, the methods most proper for the purpose in question. They flow immediately from the principles already laid down in this essay, and do not need many words to make them completely understood.

37. A work of this kind, if it is properly conducted, must be begun at the lower end of the river or canal; that is to say, at that end where their waters are discharged into the sea, or where they fall into some other greater river or canal, from whence their waters are carried off without farther hindrance. If it is a river whose bed, by being filled up with mud, sand, or other obstacles, and by being otherwise become irregular in its course, is thereby often subject to inundations, and incapable of internal navigation, the point, from which

the work must be begun and directed throughout all the rest of the channel, is from the lowest water-mark of spring tides on the shore at the mouth of the river; or even something below it, if it can be done; though this part will soon fill up again by the sand, mud, &c. which the tides cease not to roll in.

If it is a canal whose bed is to be dug anew, or one already made, which is to be cleaned and deepened from the sea shore or some large river back into the country, and where no declivity is to be lost; as is the case in all flat countries: the work must be begun, and the depth of the whole channel directed, from the low water-mark of spring tides, if the mouth is to the sea, or from such a depth in the channel of the river, if the canal falls into one, that there may be such a communication of water from the canal to the river, in all situations of the current, as may let boats freely pass from one to the other. This, of course, must also direct the depth of the floor of the last sluice towards the mouth of the canal, be it to the sea or into a river. If the bottom or floor of a sluice already constructed be too low, it will soon fill up with sand or mud, and thereby hinder the gates from opening, unless it be continually cleaned out; if, on the contrary, this floor be too high, and in a canal whose natural declivity is too little for the free current of the water, as is

generally the case in Holland and Flanders, all depth of the bed of the canal below the horizontal level of the bottom of the sluice will serve to no manner of purpose, either for navigation, or for carrying off the back-waters, but will soon fill up with mud, in spite of all means used to the contrary, except that of digging it continually anew to no manner of purpose; as is evident from the reasons given above (N^o 33. 34.).

38. Setting off from this determinate point, at the mouth of a river, or at the bottom of the last sluice upon a canal, which are to be cleaned and deepened; the work must be carried on, in consequence, uniformly throughout their whole course backwards into the country as far as is found necessary for the purposes intended. This is to be done after the following manner :

1st, One must dig up and carry away all irregularities in the bottom and sides of the bed, such as banks of sand and mud, rocks, stumps or trunks of trees, and whatever else may cause an obstacle to the regular motion of the water, and to the free passage of vessels upon it.

2^{dly}, If the declivity of the bed should be still too little to give a sufficient current to carry off the water as often and as fast as is necessary, the whole bed itself must be regularly deepened, and what is dug out from the bottom must be laid upon the sides, to render it narrower in proportion

proportion to its depth. The reason of this is evident from all that has been said.

3dly, Wherever the banks are too low to contain the stream in all its situations, they must be sufficiently raised; which may be conveniently done with what is dug out from the bed: and the whole being covered with green turf will render these banks firm and solid against the corrosion of the water. It is proper at all times to lay upon the banks what is dug from the bed, by which they are continually strengthened against the force of the current.

4thly, It is often necessary to diminish the windings and sinuosities in the channel as much as possible, by making new cuts whereby its course may approach towards a right line. This is a great resource in flat countries subject to inundations; because thereby all the declivity of a great extent of the river, through its turns and windings, may be thrown into a small space by cutting a new channel in a straight line; as may generally be done without obstacle in such countries as I am speaking of, and hereby the velocity of the current will be very greatly augmented, and the back-waters carried off to a surprizing degree, as is evident from what is said above in N^o 29.

5thly,

5thly, Wherever there is a confluence of rivers or canals, the angle of their junction must be made as acute as possible, or else the worst of consequences will arise from the corrosion of their respective streams; what they carry off from the sides will be thrown into irregular banks in the bottom of the bed. This acute angle of junction may always be procured by taking the direction at some distance from the point of confluence.

6thly, Wherever the sides or banks of a river are liable to a more particular corrosion, either from the confluence of streams, or from irremediable windings and turns in the channel, they must be secured against it as much as possible by *weirs*: for this corrosion not only destroys the banks, and alters by degrees the course of the river, but also fills up the bed, and thereby produces all the bad effects we have spoken of above in N° 33. 34. &c.

7thly, But the principal and greatest attention in digging the beds of rivers and canals must be had to the *quantity and form of their declivity*. This must be done uniformly throughout their whole extent, or so much of it as is necessary for the purposes in hand, according to the principles laid down above (in N° 29 and 30.) Conformable thereto, the depths of their beds, and of the floors of their sluices, at the mouths whereby they discharge

charge their waters, being fixed according to what we have said in N° 37. the depth of the rest of the beds, and the quantity of declivity therein, must be regulated in consequence thereof, so as to increase regularly the quantity of declivity in equal spaces the farther we recede from their mouths, and proceed towards their sources or to the part where the regular current is to take place.

If the depth and volume of water in a river or canal is considerable, it will suffice, in the part next the mouth, to allow one foot perpendicular of declivity through six, eight, or even, according to DESCHALES ^(d), ten thousand feet in horizontal extent; at most it must not be above one in six or seven thousand. From hence the quantity of declivity in equal spaces must slowly and gradually increase as far as the current is to be made fit for navigation; but in such a manner, as that at this upper end there may not be above one foot of perpendicular declivity in four thousand feet of horizontal extent. If it be made greater than that in a regular bed containing a considerable volume of water, the current will be so strong as to be found very unfit for the purposes of navigation, as will appear hereafter, when I come to investigate the quantity of declivity in several rivers, the degree of swiftness of whose currents is well known.

(d) De Fontibus et Fluviiis, prop. 49.

39. I dare boldly affirm, from the certain principles of hydrodynamics laid down in this essay, that if the above mentioned things (N^o 37. 38.) were carried into execution in a proper manner; the velocity of currents and the acceleration of motion of the waters in rivers, and in canals when their sluices are open, might be increased to any degree that can be required for opening their beds, and for preventing inundations during great rains or sudden floods: by carrying off more swiftly the great accession of water which then takes place. It would not be difficult, by these means, to increase the velocity of the current to double and triple what it is in rivers and canals, whose beds for a long space of time have been left to themselves. There is not, perhaps, a country on earth but what might be freed from inundations by these means. But it may be objected, that if all I have advised was put in execution, even in the flattest countries, the currents of rivers (for canals shut up with sluices are here out of the question) would become incommodious, if not unfit, for navigation, especially against their streams. This objection would be of weight if it was not evident that the various means which I have pointed out may be executed in whole or in part, to a certain degree, and no farther than necessary for the purposes required. But, as it is certain that a strong and regular

regular current in a river is the best of all means for keeping it open and deep, and for preventing the formation of banks in the bed by the subsidency of mud, &c. which it does not allow time to precipitate; I leave it to be considered, whether it is better to have a free and open navigation something incommoded by the strength of the current, or to have soon no navigation at all, without repeatedly digging the bed anew.

40. I shall not here enter into the mechanical part of the methods of digging and cleaning canals, rivers, and sea ports; or into any description of the machines and instruments necessary for that purpose. The subject would lead me much too far: besides all these things may be found much at length in most of the authors who have wrote upon hydraulic-architecture, such as BARATFERI, CORNELIO MEYERI, GUGLIELMINI, and a notorious anonymous French plagiarist, who has taken from MEYERI, without ever naming him, almost all that is contained in his book, published at Paris in 1693, and at Amsterdam in 1696, in octavo, under the title of *Traité des Moyens de rendre les Rivières navigables*. But the author who has treated this subject with the greatest care, and most at length, is the celebrated BELIDOR, in his *Architecture Hydraulique*, 4 vol. in quarto. To these may be added a late memoir of M. FOREAULT of

Rouen, vice-architect of the French navy, which gained the prize of the Royal Academy of Sciences and Belles Lettres of Mantua, for having given the best solution of a problem proposed by that Society in 1776, in the following terms: “ *To indicate the best and cheapest method of freeing navigable canals from banks of sand and earth formed in their beds which render them too shallow.*” This piece, printed at Mantua, by Pazzoni, in 1778, contains sixty-three pages in quarto, and is divided into two parts; the first contains the means of preventing the formation of banks in navigable canals; and the second offers divers methods for remedying them when they are already formed. For this purpose the author proposes six different machines of his own invention: the first may be employed in rivers near the sea, and subject to the ebb and flow of the tides; the second may be used in those where the waters are always nearly of the same height and velocity; the third and fourth are to be used in those places where the violence of the currents corrode the beds; and the two last serve to break up the banks of sand or earth formed in the bottom, and to carry off all heterogeneous bodies sunk in the river, which cause an obstacle to the current. It would be difficult to give a just idea of these machines without the help of the six plates which accompany the
piece;

piece; but as this production of a foreigner has been crowned in Italy, the country of all others in which, from all antiquity, the science of rivers and canals has been most cultivated, we cannot well doubt of its merit, or that it is worthy of a translation into our own language.

V. *Other considerations on the nature of rivers and inundations.*

41. Rivers flowing along plains, as well as through vallies, have naturally their beds in the lowest part of the ground comprized between the opposite hills or mountains: nevertheless, the surface of the water of a river in the midst of a plain is often higher than the surface of the grounds adjacent to the banks of the river. This proceeds from the continual subsiding of the mud, &c. brought down by the stream during floods; the waters in that case usually overflowing the banks spread themselves over the plain, where they lose a great part of the swiftness of their current, which contributes greatly to the subsiding of the mud they contain; so that the farther they flow upon the plain, the clearer they grow, and the less remains to subside. From hence the greatest precipitation of mud must be in the parts of the plain nearest the sides of the river, which in length of time will raise

these grounds above the rest of the plain. Again, the waters in the bed itself depositing incessantly a part of the mud, &c. brought down by the stream, must continually, though insensibly (for a long space of time) raise the channel and banks of the river above the rest of the plain. These causes may at last contribute to the forming of an entire new bed for the river; for as all rivers carry down in their streams more or less mud and other heterogeneous matters, which do not subside regularly in all parts alike, but must precipitate fastest where the current is slowest; there must accumulate by little and little in these parts such banks of sand and mud, as will in time hinder the current of the waters, make them reflow, and at last totally change their direction.

Canals are still more subject than rivers to have their beds raised and their currents stopped by the subsiding of mud and heterogeneous matter in different places, and especially just above their sluices; because of the sudden stagnation of the water which first begins there as often as the sluices are shut: and as there is a necessity for keeping them for the most part shut, the stagnating waters in their beds must precipitate their mud, &c. in a much greater proportion than can be done in the currents of rivers, which are in a continual motion towards the sea.

42. I call *center of the current*, or, more properly, *line of greatest current*, that *line which passes through all the sections of a river, in the point where the velocity of the current is the greatest of all*. We have seen above (N^o 25.) that if the current of a river is regular, and in a right line, its center or line of greatest velocity will be precisely in the center of the sections (N^o 6.): but, on the contrary, if the bed is irregular and full of turns and windings, the center or line of greatest current will likewise be irregular, and often change its distance and direction with regard to the centers of the sections through which the waters flow, approaching successively, and more or less, to all parts of the bed, but always in proportion and conformably to the irregularities in the bed itself.

This deviation of *the line of greatest current* from the centers of the sections through which it passes, is a cause of many and great changes in the beds of rivers, such as the following:

1st, In a straight and regular bed, the greatest corrosion of the current will be in the middle of the bottom of the bed; because it is that part which is nearest to the line of greatest current, and at the same time which is most acted upon by the perpendicular compression of the water. In this case, whatever matters are carried off from the bottom will be thrown, by the force of the current,

current, equally towards the two sides, where the velocity of the stream is the least in the whole section.

2dly, If the bed is irregular and winding, the line of greatest current will be thrown towards one side of the river, where its greatest force will be exerted in proportion to the local causes which turn it aside: in short turns of a river there will be a gyration, or turning round of the stream, by reason of its beating against the outer side of the angle; this part will be corroded away, and the bottom near it excavated to a great depth. The matters, so carried off, will be thrown against the opposite bank of the river where the current is the least, and produce a new ground, called an *alluvion*.

3dly, Inequalities at the bottom of a river retain and diminish the velocity of the water, and sometimes may be so great as to make them reflow: all these effects contribute to the subsiding of sand, earth, and other matters thereon, which cease not to augment the volume of the obstacles themselves, and produce shallows and banks in the channel. These in time, and by a continuance of the causes, may become islands, and so produce great and permanent changes and irregularities in the beds of rivers.

4thly, *The percussions of the center of the current* against the side of the bed are so much the greater as they are
made

made under a greater angle of incidence; from whence it follows, that the force of percussion, and the quantity of corrosion and of detriment done to the banks and weirs of rivers, and to the walls of buildings made therein, and which are exposed to that percussion, *are always in a direct compound proportion of the angle of incidence, of the greatness and depth of the section together, and of the quantity of velocity of the current.*

5thly, It may happen in time, that the excavation of the bottom, and the corrosion of the sides, will have so changed the form of the bed as to bring the force of percussion into equilibrium with the velocity and direction of the current; in that case, all farther corrosion and excavation of the bed ceases (N° 31.)

6thly, This gives the reason why when one river falls into another almost in a perpendicular direction, and makes with it too great an angle of incidence, this direction is changed in time, by corrosions and alluvions, into an angle much more acute, till the whole comes into equilibrium.

7thly, So great and such continued irregularities, from local causes, may happen in the motion of a river, as will intirely change its ancient bed, corrode through the banks, where they are exposed to the greatest violence of percussion

percussion of the stream, and open new beds in grounds lower than what the old one is become.

8thly, Hereupon the state of the old bed will entirely depend on the quantity of water, and on the velocity and direction of the current in the new one; for immediately after this division of the waters into two beds is made, the velocity of the current in the old one will be diminished in proportion to its less depth. In consequence thereof, the waters therein will precipitate more of their mud, &c. in equal spaces than they did before; which will more and more raise up the bottom, sometimes even till it becomes equal with the surface of the stream. In this case, all the water of the river will pass into the new bed, and the old one will remain intirely dry. It is well known, that this has happened to the Rhine near Leyden, and to many other rivers.

9thly, Hence the cause of the formation of the new branches and mouth, whereby many great rivers discharge their waters into the sea.

43. But in proportion as *a river, that has none of these obstacles in its bed*, approaches towards its mouth, we see the velocity of its current augment, at the same time that the declivity of the bed diminishes, the causes of which have been explained above (N^o 29.). It is for this reason, that inundations are more frequent and considerable

derable, and do more damage in the interior parts of a country, than towards the mouths of most rivers.

In the Po, for example, the height of the banks made to keep in the waters diminishes as the river approaches to the sea. At Ferrara they are twenty feet high; whereas nearer the sea they do not exceed ten or twelve feet, although the channel of the river is not larger in one place than in the other.

44. The mouths of rivers, by which they discharge their waters into the sea, are liable to great variations, which produce many changes in them.

1st, The velocity and direction of the current at these mouths are in a continual variation, caused by the tides, which alternately retard and accelerate the stream.

2dly, During the flowing of the tide, the current of the river is first stopped, then turned into a direction intirely contrary throughout a considerable extent; if we may believe M. DE BUFFON, there are rivers in which the effect of the tides is sensible at 150 or 200 leagues from the sea.

3dly, This state of things is a cause of a great quantity of sand, mud, &c. being precipitated and accumulated in the channel near the mouth. This continually raises and widens the bed, and at last changes it intirely into a new place, or at least opens new mouths to discharge

charge the waters at. The Rhine, the Danube, the Wolga, the Indus, the Ganges, the Nile, the Miffiffippi, and many other rivers, are instances of this.

4thly, All these effects are less sensible at the mouths of little rivers, as their currents oppose no sensible obstacle to the flowing of the tides; so that the ebb carries off again what the flow had brought in.

45. Whenever the course of a river throughout a considerable extent of country approaches towards a right line, its current will have a very great rapidity; and the velocity wherewith it runs diminishing the effect of its natural gravitation, the middle of the current will rise up, and the surface of the river will form a convex curve of sufficient elevation to be perceived by the eye; the highest point of this curve is always directly above *the line of greatest current* in the stream.

On the contrary, when rivers approach near enough to their mouths for a sensible effect to be produced in them by the flowing of the tides; and also when in other parts of their course they meet with obstacles at the sides of their channel: in both these cases the surface of the water at the sides of the current is higher than in the middle, even though the stream be rapid. In this situation of things, the surface of the river forms a concave curve, the lowest point of which, or that of inflexion, is

directly over *the line of greatest current*. The reason thereof is, that there are in this case two different and opposite currents in the river; that whereby the waters flow towards the sea, and preserve their motion therein even to a considerable distance; and that of the waters which remount, either by the flowing of the tide, or by their meeting with local obstacles, which form a *counter current*, so much the more sensible as the flowing of the tide is stronger, or as the percussive of the water is made against greater obstacles, and in a direction nearer to a perpendicular to them. From both these causes, the greater of which by far is that of the tides, the water near the sides of the channel, where the velocity of the descending stream is naturally the least (N^o 25), takes a contrary direction, and runs back in the river, while that in the middle continues to flow on towards the sea. This counter current is what the French call *a remous*.

An island in the middle of a river produces the same effect as obstacles at the sides, regard being had to the difference of situation of each.

Eddies and *whirlpools* in rivers, in the center of which there appears a conical or spiral cavity, and about which the water turns with great rapidity and sucks in whatever approaches it, proceed in general from the mutual percussive of these two counter currents; and the vacuity in

the middle is produced by the action of the centrifugal force, whereby the water endeavours to recede, in a direct ratio of its velocity, from the center about which it moves.

46. If rivers persevered always nearly in the same state (N^o 5.) the best means of diminishing the velocity of the current, when it is found too great for the purposes of navigation, would be by widening the canal: but as all rivers are subject to frequent increase and diminution, and consequently to very different degrees of velocity and force in the current, this method is liable to produce very detrimental effects; for, when the waters are low, if the channel is very large in proportion, the stream will excavate a particular bed, which, according to the irregularities of the bottom, will form various turnings and windings with regard to the principal bed; and when the waters come to increase, they will follow, to a certain degree, the directions which the bottom waters take in this particular bed, and thereby will strike against the sides of the channel, so as to destroy the banks and cause great damages.

It would be possible to prevent in part the bad effects proceeding from the current striking against the banks, by opening, at those places where it strikes, little gulfs into the land, dug in such a form and direction as that the striking

striking current should enter and circulate therein, so as to destroy, or at least greatly diminish, its velocity. This effect would be felt for a considerable way down the river.

This same method might probably be used with success against the destruction of bridges, weirs, &c. by the violence of the stream during floods. Such gulfs being dug into the outer side of those turnings in the river which are immediately above the place to be secured from the violence of the stream, would successively diminish its velocity, its force and dangerous effects, a considerable way down the river. It is true, this method might contribute to produce an overflowing of the river upon the grounds adjacent to those artificial gulfs, this being a natural consequence of the decrease of the velocity of the current in those places; and it would remain to be considered whether those local inundations, or the danger of destruction of the bridges or edifices in the river, were the lesser evil.

47. The nature of inundations, and the manner of their formation, merit a particular attention in this place.

While the volume of water in the *bed* of a river increases, the velocity of the current increases in proportion, as has been repeatedly shewn above (N^o 13. 18. 20.

23. 27. 28. 29.) But from the moment that part of this water overflows the bed, the velocity thereof begins to diminish (N^o 41.) and does so more and more, the farther it flows and spreads on the plain. So that the overflowing being once begun, it is a natural consequence, that the inundation should continue for several days; for though the volume of water brought down by the flood during that time should decrease, yet, as the quantity of what runs off decreases likewise, from the great decrease of velocity in what overflows the plains, it will continue to produce the same effect as if the volume of water coming down had not diminished, until the whole of the stream be every where contained again within the bed of the river. When that is become the case, the waters that have overflowed the plain will decrease thereon, by gradually and slowly running off, and also by evaporation, till they wholly disappear. If this was not so, we should see rivers overflow for an hour or two, and then return again within their beds, a thing contrary to general observation; for we constantly see inundations, once begun in flat countries, last for several days together, although in the mean while the rain ceases, and the quantity of water coming down diminishes. This must be the case, because as the overflowing diminishes the velocity, and consequently the quantity of water carried

carried off, it has the same effect as if a greater quantity still continued to come down.

It may not be useless to remark here, that what we have often said in this essay becomes evident from these observations on nature, as well as from the principles laid down in it; to wit, that the most direct and efficacious method of preventing inundations is by deepening the bed and raising the banks of the river.

It may likewise be observed, with regard to inundations, that if the wind blows directly contrary to the current of the river, the overflowing will be greater than it would have been otherwise, because this accident diminishes the velocity of the stream: but, on the contrary, if the winds blow in the same direction with the current of the river, the inundation will be less than otherwise, and sooner at an end; because this accidental cause augments the velocity of the stream.

VI. *On the confluence of rivers, and on the separation of the same river into divers branches and mouths, with the effects thereof upon the velocity of currents, inundations, &c.*

48. All great and long rivers receive into their beds many others of different magnitude throughout the extent

tent

tent of their course. This is evident to every one who only casts his eyes over a map. The Rhine and the Po, in particular, receive each above one hundred others great and small; the Danube above two hundred; the Wolga as many; the river of Amazons receives into its vast bed a prodigious number, some of which are five or six hundred leagues in length, and are of such a depth and breadth as would make them elsewhere pass for capital rivers. M. DE BUFFON ^(c) gives a list of the more considerable of those which fall into other great rivers throughout the known world. Many curious particulars may be seen in VARENIUS's General Geography, part I. chap. XVI. concerning rivers; but of a nature which does not enter into my plan. The works themselves are in every body's hands, and may be consulted by those who please.

This confluence of rivers is so necessary for propagating the motion of the water throughout a long course, and for renewing and accelerating from time to time its velocity, which otherwise would be too greatly diminished by the resistance of so many obstacles as they meet with in their way, that, as we have said above (N^o 28.) after Signor GUGLIELMINI, it can only be attributed

(c) Hist. Nat. tom. II. p. 75, 76.

to the infinite wisdom of the Author of Nature, in the original disposition of things.

49. We have seen above (N^o 18. 27. 28.) that the increase of a river or canal by the new waters which it receives, is *universally in an inverse ratio of the new velocity which is acquired therefrom*. If this velocity is greater, the increase of the section of the new stream will be less in proportion, and *vice versa*. It follows from hence, that it is possible for one river or open canal to fall into another river or open canal of equal magnitude with itself, and yet the section of the current in the common bed after their confluence shall be no greater than it was in each of them before their junction. It is certain that this will be the case as often as *the confluence of the two augments the velocity of the common current in the same proportion with the increase of the volume of waters*; both the greater rapidity of the current, and the greater volume of water in the bed after the junction, serving to deepen it in proportion to its breadth, will contribute towards the above effect. Another cause will likewise add thereto; to wit, that instead of the resistance from the attraction, friction, and other obstacles, in two beds, which give two bottoms and four sides, there are, after the confluence, only those of one bed, of one bottom and two sides. Moreover, the center of the section in the common bed

is farther from the bottom and sides thereof, than it is in the separate beds. All these causes, in proportion to their respective quantities, contribute to accelerate the velocity of the common stream.

50. It is not less certain, that in rivers which bring down a great abundance of water, the more the velocity and discharge thereof at their mouths are retarded and diminished by the tides, the winds, the rolling in of the sea, &c. the more will the back-waters increase in height, and endanger overflowing the inner parts of the country. This is evident, because the decrease of velocity in the current, and the increase of height of all the back-waters that are affected thereby, are in a reciprocal inverse ratio one of another (see above N° 32.)

Nature itself teaches us a method of preventing, or at least of diminishing, this effect. We see all great rivers overcharged with a vast volume of water divide, when they come near the sea, into different branches and mouths, whereby the super-abundance of their waters is discharged. This is the case with the Scheld, the Rhine, the Rhone, the Po, the Danube, the Wolga, the Euphrates, the Indus, the Ganges, the Nile, the Niger, the Oroonoko, the River of Amazons, and with almost all other great rivers.

This separation and dispersion of the too great quantity of water into several channels is one cause of their seldom overflowing the country near their mouths ^(f), because it gives a greater depression and declivity to the surface of the current, and thereby facilitates the running down of the waters from the interior parts of the country, forasmuch as their beds are every where regular and free from obstacles to their current.

51. Notwithstanding the apparent opposition to what has been said in several other parts of this treatise, I repeat again, that this division and dispersion of the waters into several branches and channels *when there is such an abundance of it as is sufficient to keep up the velocity both in the old and new channels*, augments the declivity, and thereby facilitates the running off of all the back-waters from the inner parts of the country, as far as the bed is regular and free from obstacles, according to what is laid down above (N^o 38.).

But whenever this super-abundance of waters, sufficient for keeping up the velocity in each channel nearly to what it was before the separation or divarication, shall be found wanting, it is certain, that this division and dispersion of the waters into several channels will only serve to diminish the velocity of the current in each, whereby

(f) Another cause thereof is pointed out above, N^o 43.

as much or more discharge of the water, and consequently of declivity for the running off of the back-waters, may be lost, as has been gained by the separation into different beds.

This disadvantage may be easily remedied in those new channels and mouths of rivers which are dug by hands, and have sluices placed in them at the point of separation from the original bed; for these sluices of communication need be opened only when there is a super-abundance of water in the river, sufficient to keep up the velocity in each of the channels; at other times they may be kept shut, and the waters retained in their original bed.

52. It was for this purpose, of preventing the damages proceeding from immoderate inundations, that the ancient Egyptians dug vast lakes, and made so many canals and sluices of communication between the Nile and those lakes, and from thence to the sea; that they might thereby be able to discharge the waters into those reservoirs if they came down in too great abundance, or let them off again from thence upon the land, if the quantity of the natural inundation at any time was less than what was necessary for the good of the country. By these means ancient Egypt was always master of its waters.

It is well known that it rains seldom in that country, and that the Nile by its regular inundations waters the land, by bringing down upon it the rains and melted snow from the high mountains of Abyffinia. HERODOTUS ^(g) and DIODORUS SICULUS ^(h) have left us descriptions of the immense labours of the inhabitants to govern and multiply so beneficent a river, the particulars whereof are too well known to be repeated in this place. By these means Egypt became the granary of the world for above two thousand years, and reimbursed, with immense advantage, the first expences.

RICCIOLI ⁽ⁱ⁾ assures us, that the ancient Persians did the same thing with regard to the *Euphrates*, and for the same end. He adds, moreover, *Sic ubi Cyrus Gangem in Aheos 460 dispersit, minora damna ex Gangis alluvionibus campi perpesti sunt*; but I am totally at a loss to find upon what authority he grounds this last assertion, for I never read that any CYRUS penetrated as far into India as the mouth of the *Ganges*, much less reigned so long over that country as to perform the vast work which RICCIOLI speaks of.

(g) In lib. II.

(h) Biblioth. l. II. c. 1.

(i) Geogr. et Hydrogr. l. VI. cap. xxix. p. 248, 249.

PLINY ^(*) says, with regard to the different mouths of the Po, *Omnia ea flumina fossasque primi à Sagi fecere Tbusci, egesto amnis impetu per transversum in Atrianorum Paludes, quæ Septem Maria appellantur.* These seven lakes discharged their waters into the sea by seven mouths, which PLINY names in the same place. All this was apparently done that the river might do less damage to the adjacent countries by its frequent inundations. PLINY adds, *His se Padus miscet, ac per hæc effunditur, plerisque, ut in Ægypto Nilus, quod vocant Delta.*

To these examples, drawn from ancient history, might be added many modern ones, if the things in question had need of further proofs. Thus, both nature and the experience of a long series of ages teach us, that the separation of a river into several beds, by new branches and mouths, is a means of diminishing inundations in the inner part of the country; but that this takes place only when there is a sufficient abundance of water in the river to fill the new beds and channels so far as to prevent the velocity of the currents therein from being notably diminished from what they were before the division.

(*) Hist. Nat. l. III. cap. xvj.

S E C T I O N II.

Laws of the meeting of opposite currents, with the application of them to sluices.

53. When two equal currents of homogeneous fluids meet in opposite directions, there is first a swelling and rising up of them at the point of rencounter; then follows a revulsion and counter current of each equally back again, so as to bring the whole to an equilibrium.

54. If the two opposite currents are unequal, either in force or in quantity, or in both, there will still be a swelling and revulsion of each back again, but it will be diminished in the greater current, and augmented in the lesser, by the quantity by which the one surpasses the other; and the point of rencounter of the two currents will have a slow and progressive motion in the direction of the stronger, the degree of velocity thereof being always in a direct ratio of the force and quantity of the one above the other.

55. If the fluids in opposite currents be not homogeneous, as is the case between sea and river water, that which has the least specific gravity will swim upon the other, and continue to follow its first direction, until
such

such time as the heavier fluid shall have communicated its motion to all the parts of the lighter. But the lighter fluid will not lose its former motion and direction at once, but *in a decreasing series, the law whereof will vary according to the greater or less difference of specific gravity in the two fluids*, until the whole of the lighter has acquired the velocity and direction of the heavier which buoys it up.

The *time* and *space* required for a greater current of salt water to communicate its motion and direction to an opposite one of fresh water will be but very little, since they differ in specific gravity only $\frac{3}{73}$ parts that the salt is heavier than the fresh. It would require much greater between water and oil, and still much more between quicksilver and oil, and so on. The elements for determining them in every case might be found by a proper number of experiments.

56. Let the two currents be equal or unequal in force and velocity *but nearly of the same specific gravity*, if we should suppose at the same time that their surfaces are not upon a level, but that the one is higher than the other (as is constantly the case in all sluices that open to the sea, except at the moment when the surface of the tide is upon a level with the surface of the water in the canal behind the sluice); this circumstance entirely changes both the case and the effects. It is certain, on
this

this supposition, that the overplus of velocity and elevation in the higher current, though it should be the lesser, will make the waters in the lower and greater current reflow upon themselves until they come to a level and equilibrium with those in the upper current; since these, by the laws of universal gravitation, cannot flow back from a lower to a higher level, but must descend according to the declivity of the surfaces. If the currents are of *very different specific gravities*, they will come to an equilibrium according to the law laid down above (N^o 55); but their greater or lesser quantity and velocity will produce little or no effect in this case.

57. Now as the running of two currents in opposite directions, after their rencounter, and beyond the limits laid down above (N^o 55.), is incompatible with, and contradictory to, the laws of nature, and consequently impossible; we may draw this useful conclusion, which becomes important during inundations, and especially during the annual overflowing of the low grounds in flat countries; to wit, that if the sluices next the sea against which the tide flows be shut only a quarter of an hour before the *flood* has risen to the level of the water in the canal, not a drop of salt water can enter the said canal, nor even into the sluice itself; because both the progressive motion of *the point of rencounter of*

the two currents, and the *over-swimming* of the fresh water upon the salt, will be always without the sluice and towards the sea, so long as the surface of the tide is below the level of the water in the canal. Many sluice masters, for want of knowing or considering this, are accustomed to shut their gates next the sea a little after *half flood*; under the pretence of preventing by this means the salt water from getting into the canal, and communicating thereby with the waters that overflow the low grounds in many places during winter, which would be of great detriment to the soil. Through this false persuasion, they lose no inconsiderable part of that time every day, which they might safely employ in drawing off the waters which overflow and incommode low and flat countries almost every winter and rainy season, as is the case in the Dutch and Austrian Netherlands.

S E C T I O N IV.

Experiments to determine the different velocities, in different depths of water, of the same floating body moved uniformly by an equal force.

58. It is well known already, that for facilitating or retarding the motion of boats, &c. in canals, the different

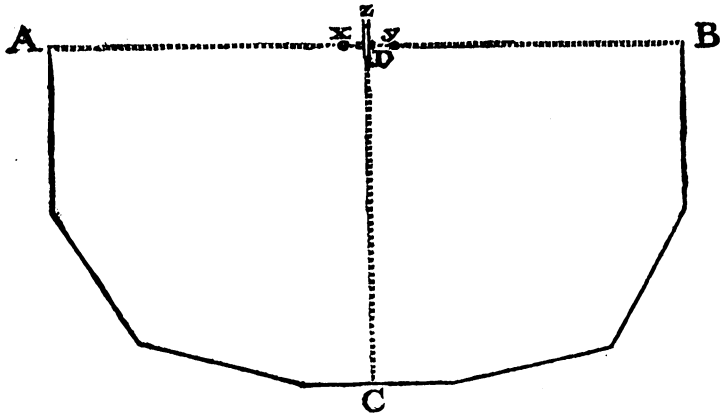
depths of the water, above that simply necessary to keep them afloat, is a thing not at all indifferent. Dr. FRANKLIN has already treated this subject, though perhaps not with sufficient accuracy, in a letter to Sir JOHN PRINGLE, written in the year 1769. He proves, however, that it is universally known among people accustomed to work boats on canals, that there is a considerable difference in the swiftness of their motion according to the greater or less depth of the water therein; and that the water being low is of itself sufficient to retard the motion of a boat, without the keel thereof rubbing against the bottom of the canal. The reason he assigns for it is evident; for a boat cannot advance its own length in a canal without displacing a quantity of water equal in mass to the space which the boat occupies under the surface of the fluid. The water so displaced must retrograde, and pass under, and to the right and left, of the boat: so that the less depth and breadth of water there is in the channel, the more in proportion it must rise up and weigh against the boat, and the more difficulty it must find in passing under and along side of it, and necessarily must retard so much the more the motion thereof. The result of Dr. FRANKLIN's experiments on this subject may be seen in the letter above mentioned.

59. Mr. NEEDHAM, Director of the Imperial Academy of Sciences at Bruxelles, being of opinion that Dr. FRANKLIN'S experiments were made upon too small a scale to draw any very exact inferences from them, desired me, at the beginning of the year 1775, to make a new set of experiments upon a much larger scale and with all possible exactness; I did accordingly, and shall here give a short description of them.

I got made, by the ship carpenters of Nieuport upon the Coast of Flanders, an exact model of a bilander, answerable in all its proportions to those used in the Low Countries. Its length was thirty-nine English inches, its breadth nine inches and a half; and its depth nine inches. Its form both within and without exactly represented that of a bilander. At each end of it was fastened perpendicularly a round and polished rod, ten inches and a half in height above the sides of the boat.

I got made likewise a wooden canal, twenty feet in length, thirty-seven inches in breadth, and sixteen inches in depth; a section of which is represented in the following figure:

This



This form is that of the excavation of the canals in the Low Countries, and approaches to that of the natural beds of rivers inasmuch as they are regular. Here $AB=37$ English inches, $DC=16$ inches; and the length of the whole canal, as we said before, was twenty feet. z represents the section of a pulley fixed at one end of the canal, upon which passed a small cord, one end of which was tied to the round rod at the fore part of the boat, and at the other end was a piece of lead which weighed eight ounces. This served for an equable force to give an uniform motion to the boat throughout all the experiments. x and y are sections of two other cords stretched parallel to each other at about one inch and a half distance, and reaching from one end of the canal to

the other. The two round rods fixed at the ends of the boat, moving within these parallel cords, served to make the boat move in a right line in the middle of the canal, without running against either side, which it would have done without this precaution. The canal itself was upon an exact level, and one end of it, where the pulley was fixed, rested upon the side of a well twenty-three feet deep, twenty of which were above the surface of the water; which gave sufficient space for the free and uniform descent of the lead-weight and cord running over the pulley, as they drew the boat from one end of the canal to the other.

Latts, exactly divided into inches, were nailed against each end of the canal within, to mark the different depths of the water in it according as it should be augmented or diminished. The outsides of the little boat, from its keel upwards, were likewise divided into inches. In the inside of the boat was a quantity of sand sufficient to sink it to six inches deep in the water. The common loaded bilanders in the Low Countries usually draw six feet of water.

Thus the form of the wooden canal, together with its breadth and depth, and the form and dimensions of the little boat, together with the depth of water it drew by means of its ballast of sand, exactly corresponded with those

those in the real canals and bilanders in the Low Countries, an inch in the one answering to a foot in the other.

Cloſe to the canal, and out of the way of all wind, was ſuſpended a pendulum of fine waxed thread, to prevent the variations of the atmoſphere from altering its length, which from the point of ſuſpenſion to the center of gravity in the lead was $39\frac{1}{5}$ Engliſh inches, ſo that its iſochronic vibrations were exactly ſeconds of time.

60. It was neceſſary, in order to render the experiments exact, that they ſhould be made at a time when the air was perfectly calm; for the leaſt breath of wind, during the motion of the boat, cauſed great variations and irregularities in them, which it was abſolutely neceſſary to prevent, in order to be able to deduce any exact reſults from them. On the contrary, in a perfect calm, the times of the paſſage of the boat, from one end of the canal to the other, were exceedingly regular, as may be ſeen from the table of experiments which I give below.

By means of the pendulum I was able to meaſure the times of paſſage of the boat along the canal, in all the different depths of water, to a third or even to one quarter of a ſecond. The boat being held faſt againſt the back end of the canal by the hand of an aſſiſtant, and then let go, it was eaſy for me to perceive the precise
instant

instant of the beginning of its motion, to let go the pendulum at the same moment, and to count its vibrations till the instant that the boat struck with an accelerated force against the fore end of the canal. As to the weight of eight ounces suspended at the end of the cord, and which served as a *moving force* to draw the boat along the canal, it was just as much as sufficed to counter-balance the cord, and to put the boat in motion; less weight than that would do neither: therefore I was obliged to use so much, notwithstanding the considerably accelerated motion it gave to the boat.

This is the whole mechanism of the instruments I used for the experiments in question, and such were the precautions I judged it necessary to use for making them with scrupulous exactness.

In the following table, which consists of twelve columns, the first of them contains the different depths of water at which the experiments were made; the ten following ones contain ten different experiments made at each depth of water in the canal; and the twelfth or last column is the reduction of the ten others to a mean proportional or mean result of the whole, which is in *seconds of time*.

Table

Table of experiments made to ascertain the times of passage of the boat along the canal, or its different degrees of velocity, in different depths of water.

Depths of Water.	1st Exp.	2d ditto	3d ditto	4th ditto	5th ditto	6th ditto	7th ditto	8th ditto	9th ditto	10th ditto	Mean Results.
Inches 15	14½	14¾	14¾	14½	14½	14½	14¾	15	14½	14½	14½
14	14¾	15	15	15½	15	15	14½	14½	15	15½	15
13	15½	15½	15½	15¾	15½	15¾	15¾	15½	15¾	15¾	15½
12	16½	16	16	15½	16	15¾	16	16	16	16	16
11	16¾	17	17	16½	16¾	17	16¾	16¾	16¾	16¾	16¾
10	17	18 wind	17	17½	17	17½	17½	17	17	17½	17½
9	18½	18½	18	19	19	18	18	18¾	18	18¾	18½
8	20	20	20½	20	20	20	20	19½	20	20	20
7	23½	23	22½	23	23	23	23½	23¾	23	23	23
6½	In the experiments made with this depth of water, the boat often touched the bottom.										30 by supposition

It may be observed with regard to the last column of the above table, which contains the mean results or mean quantities of time which the boat takes to pass from one end of the canal to the other in different depths of water, that it is given for the sake of destroying those little differences which are inevitable in practice; and it

threws, as nearly as possible, what the true time of passage ought regularly to be when nothing happens to disturb it.

It is also highly worthy of remark, that the *mean results* contained in this last column form a *series of numbers regularly increasing as the depths of water, wherein the respective experiments were made, regularly decrease*; so that *the different velocities of the floating body are in an inverse ratio of the respective depths of the water in which it floats with an equal impulsive force, and that according to the law of the above series*. This, perhaps, may furnish elements to calculate, pretty near the truth, the different velocities of vessels upon canals and rivers with different depths of water in all other cases whatsoever. As to the conclusions to be drawn therefrom in practice, and in the common uses of life, they are too obvious to need mentioning here.

S E C T I O N V.

On the quantity of declivity in rivers.

62. Abstracting from all resistance and friction, fluids, such as water, descend upon planes let them be never so little inclined towards the center of the earth: and the

velocity of descent increases in a compound ratio of the increase of the mass of water, and of the greater declivity of the plane which serves for its bed (N° 13.).

63. Water, though unaffected by any compression or impulsion from above, cannot remain immovable in any bed whatever except that which makes a curve perfectly concentric with the *terrestrial curve*; but in this, being every where equally affected by the force of gravitation, it will remain without motion any way.

64. It follows from hence, that the springs and sources of all rivers must be at a greater distance from the center of the earth than one semi-diameter thereof, which is terminated at the surface of the sea; without which the waters could not run to the river's mouth.

65. Therefore, the absolute elevation of the surface of rivers is continually diminished as they recede from their springs, because of the necessary declivity of the beds of rivers towards the center of the earth; for without some degree of this declivity the waters could not run at all, as has been said above (N° 62. 63.).

66. The declivity of the beds of rivers cannot be a right line making a rectilinear angle with that horizontal plane which, being continued, would intersect their respective sources; but, if it is regular, it must be a curve which differs very little from that of the earth's surface,

and this, if the direction is in the parallels of latitude due East and West, is *spherical*; but in all other directions it is a portion of an *oblate ellipsis*, on account of the earth's being a spheroid compressed by its axis. Now the horizontal plane which continued passes through the springs of rivers, is always a tangent to the curves of their beds at the point of inflexion, inasmuch as these are regular.

67. The quantity of absolute declivity from the spring in any determinate part of a river, is that perpendicular line drawn from the point of greatest current in that place till it meets *the curve concentric to the earth's surface* which passes through the river's source. The declivity of the bed below the spring is had by taking the same perpendicular from the bottom of the bed; as that of the river's surface is by taking the perpendicular from thence.

68. If a plane be extended horizontally every way from the point of tangency to the earth's surface, or from the point where it is perpendicular to any radius of the earth, water will run from every other part of the plane towards that said point which is nearer to the center of the earth than any other point in the whole plane.

69. The depression of *the curve of a river's bed*, below *the concentric-terrestrial-curve which intersects its source,*

source, being only 250 fathoms perpendicular in a course of 500 leagues, it will be sufficient to give a notable current in a regular bed throughout all that extent of river, as appears from what we have said above (N° 38.). But the depression of *the curve of the river's bed*, below *the horizontal plane which is a tangent to its source*, in this same extent of course, is not less than ninety leagues perpendicular, being always *the secant of the arc of the river's extent minus a radius of the earth in that point*.

70. It follows evidently from the above principles (N° 62—69), that the declivity and velocity of a river are less in proportion as the bed approaches nearer to being concentric with the curve of the earth's surface.

71. I shall now apply the principles laid down to determine, as near as possible, the real quantity of declivity in different rivers, making use of what is already known from experiments and actual mensuration to determine the same in all others by the comparison of the different degrees of velocity in their respective currents.

72. It is the general opinion of most of those who have examined this subject^(k), that rivers and canals which have less than one foot of declivity in 10,000 feet of course, will have very little current, unless it be by means of the great abundance of their upper waters

(k) Vide DESCHALES, de Font. et Fluv.

which give motion to those before them by their weight and impulsion. Without this the resistance proceeding from the bottom and sides of the bed, and from other accidental obstacles (N° 21.) would equal, if not surpass the ordinary causes of acceleration (N° 20) so as to diminish continually the motion of the waters, and at last render them almost stagnant (N° 22.). But nature has prepared remedies against this, as we have seen above (N° 28. 29. 48.). What RICCIOLI ⁽¹⁾ says of the *Po*, in that part of its course next its mouth, is perfectly conformable to this theory: “ Sic Padus, qui a Pago *Ostellata* “ vocato, usque ad *Adriaticum*, intervallo milliarium cir- “ citer 70, non habet libramentum majus 13 aut 14 “ pedum, ita ut singulis milliaribus ne 3 quidem uncia “ declivitatis obveniant; unde *Paduse*, potius instar stag- “ nantibus aquis, incertissimus esset ad defluxum cursus: “ impetu tamen impresso à 30 et amplius fluminibus “ aut torrentibus se in illam exonerantibus, etiamque a “ nativæ pondere aquæ ex superioribus et altioribus “ prope Alpes alveis decurrentis, velocitatem maximam “ acquirit.”

73 From many observations and trials which I made for this purpose in the years 1773 and 1774 upon the river *Iprelee* in Flanders, which comes down from the

(1) *Geogr. et Hydrogr.* J. VI. c. XIX. p. 215. edit. Bonon. 1661.

city of Ipres and falls into the sea at Nieuport, having a *very moderate current* when the sluices upon it are open, I found its *mean declivity* to be nearly three fathoms four feet and eight inches in 20,000 fathoms of extent of its course, or very nearly one foot in a measured English mile. I say its *mean declivity*, because from what has been said above (N^o 13. 27. 28. 29.) it is plain, that a greater or less quantity than ordinary of water in it will add to, or take from, something thereof; but the declivity in each part of its bed is nearly uniform.

As the sources of this river, and those of the Iser which joins it at Fort Knock, four leagues from Nieuport, are in the higher grounds of Flanders towards Houthem, Mount Kemele, Swaertberg, Catfberg, and the other hills as far as Mount Cassel; and as the rest of their course is in a flat country with a very small descent towards the sea, the declivity thereof may be taken as a *mean* between that of the other rivers and canals of Flanders: the *artificial canals* will have less, not above a six or seven thousandth part of their extent, or one twelfth of an inch in each eight fathoms: the rivers Lys and Escaut, before they fall into the flat country, something more, after which they may have about the same, or even something less between Ghent and Antwerp.

This

This quantity of three fathoms four feet and eight inches of declivity in 20,000 fathoms of extent, gives the proportion of the declivity to the extent as 1 to 5292, which is one line or twelfth part of an inch in $6\frac{1}{8}$ fathoms, and two feet seven inches in one French league of 2283 fathoms. Now the measured English mile containing 5280 feet, this proportion of $\frac{1}{5292}$ approaches so very near to one foot of declivity in every measured mile of extent, that I shall every where reduce what I call the *mean declivity* to that quantity, as a standard wherewith to compare the rest.

74. In canals, all whose sluices and vents have been kept shut a sufficient time to render the water stagnant throughout their whole length, there cannot be allowed above an inch or two of declivity for each mile in length, on account of the water that unavoidably runs off through the chinks of the doors of sluices, drains, &c.

75. According to the observations of the Abbe CHAPPE D'AUTEROCHE ^(m), the floor of the Hall of the Royal Observatory at Paris is forty-five fathoms three feet and five inches French above the level of the sea at the mouth of the Seine. According to the Abbé NOLLET, this same floor is forty-six fathom above the level of the Ocean, and only forty-five fathoms above the level of the Mediter-

(m) See Relation de son Voyage en Sibirie, tom. II. p. 406, 407. 444.

anean sea. Again, according to the above Abbé CHAPPE, the said floor of the Observatory is elevated twenty-four fathoms one foot and ten inches above the level of the river Seine at Paris; therefore the level of the Seine under the Pont Royal at Paris is twenty-one fathoms one foot and seven inches above the level of the Ocean; and such also is the quantity which Mess. CASSINI have given, from their own observations and experiments, for the mean height of the Seine at Paris above the level of the sea.

Now the course of the Seine from Paris to its mouth at Havre de Grace, by following all its turns and windings, is about 90,000 fathoms in length; therefore $\frac{90,000}{21.1.7} = 4232\frac{1}{2}$ fathoms of extent for one fathom of declivity in the river Seine, or one line in $4\frac{2}{3}$ fathoms, and consequently the proportion of its declivity to its length is as one to $4232\frac{1}{2}$. It is to be observed, that the bed of the Seine is deep, and its current considerably strong.

76. By similar observations and actual levellings made upon the river Loire by M. M. PICARD and PITOT ⁽ⁿ⁾, the declivity thereof in proportion to its length is found to be as one to 3174, which is one line in $3\frac{2}{3}$ fathoms. Notwithstanding this great declivity of the bed of the Loire it is observed, that the velocity of the water therein,

(n) See Memoires de l'Acad. Royale des Sciences de Paris, pour 1730.

compared to that in the Seine, is less than it should be in proportion to their respective declivities, which is very justly attributed to the much greater depth in proportion to the breadth of the Seine, above what is found in the Loire. This last river is remarkably broad, and so shallow that in many places it is hardly navigable for boats. Now this, according to the principles laid down above (N° 27, &c.) must very much diminish the swiftness of the current, which it should otherwise have from the great declivity of its bed. In confirmation of this it is moreover observed, that in great falls of rain, which equally increase the volume of water in both these rivers, the velocity in the Loire augments in a much greater proportion than it does in the Seine; and this observation is likewise conformable to the principles above laid down (N° 12. 28. &c.).

77. The river Doux, after passing by Befançon, falls into the Saone above Chalon; the Saone joins the Rhône at Lyons. This river, from Befançon to its mouth in the Mediterranean sea, is one of the most rapid in the known world: the velocity of its current is at least double to that of the Seine or Loire, and its course is almost in a straight line. The difference of elevation of this river at Befançon, above that of its mouth in the Mediterranean sea, after a course of about eighty-six French leagues, has been
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been found, by a long series of barometrical observations, to be about seventy-five fathom^(a), which gives the proportion of the declivity to the extent as one to 2620, or about one third of a line to each fathom. This is double the *mean declivity* of the rivers in Flanders; but the velocity of the current in the Rhone is at least triple that in the others (N° 29.).

78. From the above *data*, got from observations and actual mensuration, and from many others of the same nature too long to mention here, we may deduce the following table of comparative proportions between the declivities and velocities in different kinds of rivers.

(a) See Cours de Physique de Paris, tom. II. N° 740.

	Rates or classes of rivers and flowing waters.	Comparative degrees of the mean velocities of currents	Seconds of time wherein currents run 20 fathom.	Fathoms run by the current in 1 minute of time.	Ratios of declivity compared with horizontal length.	Fathoms of length for each 1 st inch of declivity.	Distinctive attributes of the various kinds of rivers and flowing waters.
1	0	0	0	0	$\frac{1}{10000}$	14	Channels wherein the resistance from the bed, and other obstacles, equal the quantity of current acquired from the declivity; so that the waters would stagnate therein, were it not for the compression and impulsion of the upper and backwaters.
2	$\frac{2}{3}$	180	$6\frac{2}{3}$	7000	$\frac{1}{8000}$	8	Artificial canals in the Dutch and Austrian Netherlands.
3	1	120	10	$\frac{1}{3300}$	$\frac{1}{3300}$	6	Rivers in low and flat countries, full of turns and windings, and of a very slow current, subject to frequent and lasting inundations.
4	$1\frac{1}{2}$	80	15	$\frac{1}{4000}$	$\frac{1}{4000}$	$4\frac{2}{3}$	Rivers in most countries that are a mean between flat and hilly, which have a good current, but are subject to overflow: Also, the upper parts of rivers in flat countries.
5	$2\frac{1}{2}$	55	$21\frac{2}{3}$	$\frac{1}{3100}$	$\frac{1}{3100}$	$3\frac{2}{3}$	Rivers in hilly countries, with a strong current, and seldom subject to inundations: Also, all rivers near their sources have this declivity and velocity, and often much more.
6	3	40	30	$\frac{1}{2800}$	$\frac{1}{2800}$	3	Rivers in mountainous countries, having a rapid current and straight course, and very rarely overflowing.
7	5	24	50	$\frac{1}{2000}$	$\frac{1}{2000}$	$2\frac{1}{3}$	Rivers in their descent from among mountains down into the plains below, in which places they run torrent-wise.
8	8	15	80	$\frac{1}{1700}$	$\frac{1}{1700}$	2	Absolute torrents among mountains.

I should think it quite superfluous to give any explanation of a table so clear and intelligible as the above; and shall only remark upon it that *the comparative degrees of the mean velocities of the respective currents* in the second column are the result of observations and experiments, the method of making which has been given above (N^o 26.): but as the velocity of rivers is very different in different seasons of the year, which augment or diminish greatly the mass of waters in their beds, *a mean* has been kept to, as much as possible, in the above table.

By taking the degree of velocity of the current in any river, a thing so easy to be done; and observing its other characteristics as laid down above under the title of *distinctive attributes*, it will be easy to judge very nearly of *the quantity of declivity in that part of the river*.

79. After carefully comparing what has been said in the relations of travellers, and in the best treatises of geography, upon the principal rivers in the known world, I should be inclined to class them in the following manner, particularly entreating at the same time that my opinion about it may be regarded as simple conjecture, which I leave to be rectified by those better acquainted with the matter than it is possible for me to be.

Under the first rate or class in the above table I should put that part of the channel of most great rivers which

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is in extensive plains next the sea; with regard to *the declivity thereof alone*, but not at all with regard to the velocity of the current there, which is often very great from the compression and impulsion of the upper waters, as has been repeatedly shewn above must be the case (N° 29. 38. 43. 72.).

Second rate or class. Most artificial canals in flat countries, made for the use of navigation; especially those in the Dutch and Austrian Netherlands.

Third rate or class. The river Trent; the Scheld and the Lys below Ghent; the Isere and the Iprelee below Fort Knock in Flanders; many rivers in the territories of Bologna and Ferrara in Italy; the river Meander in Natolia; and innumerable others in flat countries.

Fourth class. The Thames; the Lys and the Scheld above Ghent in Flanders; the Senne, the Dyle, and the Demmer, in Brabant; the Seine and the Somme in France; the Nile and the Niger in Africa; the rivers of St. Lawrence below Lake Ontario, the Oronoko, the river of Amazons, and the rivers of Paraguay, in America.

Fifth class. The Severn and Ouse in England; the Loire and Garonne in France; the Tagus, the Guadiana, and the Guadalquivir, in Spain; the Po and the Tiber in Italy; the Meuse, the Rhine, and the Elbe, in Germany; the Weisfel, the Neister, the Bog, and the Nieper, in

in Poland; the Don and the Dwina in Ruffia; the Amur or Saghalien in Tartary; the Yellow and Blue Rivers in China; the rivers of Cambodia, Ava, and Ganges, in India; the Euphrates; the river Zaire in Congo; the Miffiffippi.

Sixth clafs. The Rhone in France; the Ebro and Douro in Spain; the Danube; the Wolga; the Irtifch and Oby, the Jenefea and Lena, in Siberia; the river Indus; the Tigris; the Malmiftra in Cilicia.

Seventh clafs. In this clafs can only be enumerated thofe parts of rivers where they defcend from among mountains into the plain country below; as alfo fome rivers paffing through the midft of mountains.

Eighth clafs. To this clafs belong all torrents among mountains; fuch, for example, as the Bourns in the Highlands of Scotland are defcribed to be.

S E C T I O N VI.

A general and eafy method of taking levels through large extents of country where rivers pafs; and dfo of computing the heights of interior parts of continents, whofe the furface of the fea.

80. After all I have faid hitherto in this effay, and particularly in the foregoing fection, what I am about to lay

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lay down under this last head of it must appear very plain and easy. I am very far, however, from giving the methods I am going to propose *for taking the levels through whole countries and continents as far as rivers extend*, as strictly exact; I know very well that it is next to impossible they should be so, considering the continual variations in the declivities of rivers, and in the velocities of their currents in different parts, as also the impossibility of knowing the exact length of their course through all their turns and windings I only give them therefore as a general and easy method of computing the relative heights of countries without deviating much from the truth, which, perhaps, is all that may be necessary for the consideration of the natural philosopher. At all events, they may be of some use, for this end, in so many parts of the earth through which rivers pass, and where no barometrical observations, or any others whatever, for taking heights above the sea, have been, or perhaps ever will be made. They may also be found useful in taking the levels through a large extent of flat countries where regular canals and rivers pass, and where the difference of elevation is too small to be observed by the barometer, and where also the taking them through so great an extent by the common methods of levelling would be much too expensive for the purposes required. Now in

this last case I have found, by experience, that by the method I here propose the difference of heights may very easily be found, and that very near to the truth.

For this end it may be proper to premise a few necessary considerations and precautions to be observed in making use of the method I here propose. They would easily occur to any one who considers the principles whereon it is grounded; but to save trouble I shall put them down in a few words.

1. The first is, that a particular attention must be had to the quantity of water actually in the river at the time of the operation, so that according as the greater or less quantity thereof may augment or diminish the velocity of the current, allowance may be made conformable thereto in determining the quantity of declivity from the degree of velocity.

2dly, Observing this precaution throughout the whole river, or all that part of it wherein we want to find the difference of elevations, we must next endeavour to determine, as near as possible, by the principles laid down in the last section; *all the variations of declivity from the variations of velocity within those limits, and also the exact length and quantity of each.*

3dly, The same attention must be had in taking the difference of heights by canals, while their sluices and,

communications are kept constantly open, so as to effectuate a compleat natural current throughout the whole extent thereof; for in this case they are no other than rivers, and their waters follow the same laws of motion.

4thly, But in canals which are shut, and their waters kept up by sluices so as to render them nearly stagnant, the practice of this method will be different from what it is in rivers and open canals: for in this case there cannot be allowed for the declivity of the surface of the water from sluice to sluice above one inch, or two at most, in each mile of length, according as there may be fewer or more accidental drainings of the water in it (N^o 74.).

Again, as it may happen, in taking the levels of countries by the means of artificial canals, that the water in different parts may have different directions, attention must be had to *add* or *subtract* respectively the total declivity of each.

Moreover, it almost always happens, in canals where the sluices are shut, that the water on the two sides of each sluice is of a very different height, the back waters being kept up, while the lower are run off to a certain point; but in sluices next the sea, the tide against the outer gates is sometimes lower and sometimes higher than the water in the canal above. In all these cases, the difference of height must be exactly measured, and the quantity

quantity respectively added or subtracted in the account of the levelling.

5thly, After this it is necessary to determine, as nearly as possible, the length of the canals and rivers through all their turns and windings, and throughout the whole extent of country in which we want the difference of elevations. This may be done by an actual mensuration, or by the general opinion of the inhabitants of each part of the country, which, being founded upon the long and continually repeated experience of an infinity of people, will be found to differ very little from the truth, *attention being had to the quantity of their nominal measures*; even the errors in *more* or *less* will nearly compensate each other; or, finally, in great extents it may suffice to compute them from good geographical maps.

6thly, This being done by one or the other of these methods, it will be easy, from the quantity of declivity before determined for each part in particular, to find the whole quantity of declivity throughout the whole extent of country measured, or from any one part thereof to any other along the rivers or canals in question, which are supposed to be continued without interruption from one place to the other. If to this be added the relative height of the country in each place compared with the level of the water in the part of the river or canal next to each,

we shall have very nearly the difference of elevation of those two parts of the country. And thus the levels may be taken from the sea through any extent of country, nay even through whole continents, as far as rivers or canals extend without interruption. Cataracts themselves, such as those in the Nile and in the river of St. Lawrence, need not hinder the operation, since we have only to take the respective heights from which they fall into the account as we do in common sluices, and allow for the increase of velocity produced by them in the current of the river above and below the places where they exist.

82. Although I do not pretend to equal this method (of finding the difference of heights in countries) *for exactness* to the levels taken by actual mensuration, or to those found by a long series of nice barometrical observations; yet it must be allowed, that it is free from many inconveniencies, and accompanied with many conveniences, which the others are not. It may be easily carried through great extents of country, where the other methods cannot be put in practice, on account of the expence or time required; and this may be done with very little trouble, and perhaps with sufficient exactness to answer all the purposes of the natural philosopher in his considerations on the globe we inhabit. Although the method of taking heights by barometrical observations

observations is highly useful, and among mountains (where mine can be of little or no service) far preferable to every other hitherto discovered; yet it will easily be acknowledged by every one who is acquainted with what M. DE LUC^(m), Sir GEORGE SHUCKBURGH, and Colonel ROY⁽ⁿ⁾, have done upon this subject, that the greatest attention to an infinity of varying circumstances, as well as the greatest nicety and exactness both in the instruments and in repeated observations, are necessary if we would come at the truth thereby.

Again, the method of taking the difference of heights by the quantity of declivity in rivers requires no attention to the curvature of the globe, an object (as every one knows) infinitely too considerable to be neglected in the common method of levelling; as are also the great and varying refractions of the visual rays so near to the surface of the earth as they must be taken in the practice of that method. The quantity of effects and of errors in the visuals proceeding from this last cause must be very different at different times, as it depends wholly on the greater or less density, on the greater or less quantity of vapours suspended in the lowest part of the atmo-

(m) See his work on Barometers and Thermometers, in two vol. quarto.

(n) See the learned and curious treatises of these two gentlemen in the Philosophical Transactions for 1777.

sphere, the state of which seldom remains long the same. Now it is no easy matter either to determine the quantity of these exactly, or to calculate the effects and errors in the visual rays proceeding therefrom, which yet must be done to come at the truth by the common method of levelling; whereas, in the method I propose, no such considerations are necessary, as is evident from the nature of it.

But this is more than enough on a method so obvious and easy; I shall now give a few examples of it, and thereby conclude this essay, already perhaps much too long.

83. Supposing the length of the Scheld, between Antwerp and Ghent, following all its meanders, to be forty measured English miles, as it is reckoned nearly to be; and supposing the length of the said Scheld between Ghent and Tournay to be fifty of the same miles; and that of the Lys from Ghent to, Commines, where it approaches nearest to the city of Ipres, forty-six miles; it is required to know the respective differences of elevation between all these places.

It may be found above (N^o 78. 79.) that the river Scheld, between Ghent and Antwerp, has not above one foot declivity in each mile of its course; and that the

Scheld and the Lys, above Ghent, have about one foot declivity in each four thousand feet of length.

According to this, the surface of the Scheld in Ghent is about forty feet higher than it is at Antwerp; and at Tournay it is sixty-six feet higher than at Ghent, and one hundred and six feet higher than at Antwerp. So also the surface of the Lys at Commines is sixty-one feet higher than at its junction with the Scheld in Ghent, and one hundred and one feet higher than the same at Antwerp. From hence it may be deduced, that the Scheld at Tournay is about five feet higher than the Lys at Commines, through twenty-five miles of interjacent country.

84. Suppose it be required to find the difference of height between the surface of the Lys at Commines and the surface of the canal at Ipres which falls into the sea at Nieuport on the Coast of Flanders. The distance between Ipres and Commines is nearly seven measured miles, through which there is no communication by water; but there is one a great way round, which therefore, for the purpose required, must be followed through all the differences of elevation comprized therein, *viz.*

Descent towards the sea at Nieuport on the Coast of Flanders.

	Fect.
Total declivity of the Lys from Commines to Ghent, 46 miles, - - - - -	61
Declivity of the canal from Ghent to Bruges, when the fluices are shut, - - - - -	1
Difference of height of the water in the aforefaid canal above that in the canal from Bruges to Ostend, which two communicate together by fluices, - - - - -	8
Declivity from Bruges to Plaschendahl where the canal of Nieuport joins that of Ostend, - - - - -	6
Total declivity from Plaschendahl to Nieuport, in- cluding the difference of surfaces in two interme- diate fluices, - - - - -	8
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Total declivity, all one way, from Commines to Nieuport, about 95 miles, - - - - -	84
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Ascent

too far into the regions of conjecture; but as such mistakes as these are no ways prejudicial to my fellow creatures, to whom I wish to be useful, and as they may give occasion for others to rectify them, and so lead them to a subject which otherwise, perhaps, they might never have attended to, I shall hope for indulgence from all those who wish well to humanity and to useful knowledge.

from what was found by actual levels made from the Lys to Ipres by the French engineers during the time that LEWIS the XIVth was master of the country, when there were proposals for opening a canal from the one to the other.

85. I shall venture to carry my conjectures still farther, and grounding them upon the principles laid down above (N^o 78. 79.) I shall take a general view of the elevations of continents along the course of the principal rivers in the known world. I cannot, however, repeat too often, that I give this as a matter of mere conjecture and curiosity. It has not been, nor ever will be, in my power, or in that of any other particular person whatsoever, to follow the courses of all the rivers mentioned in the ensuing table from their mouths to their sources. All that can possibly be done on this head, is to examine the relations of voyagers and geographers concerning each river as far as it is known, and to reduce it by that means within the compass of the hydrometrical principles laid down in this essay. This is what I have done as far as I could; and therefore, allowing that I make great mistakes therein, yet I do not think that I merit much blame on that account, as I have done what I was able to do. If I am blame-worthy, it is for having launched out

Ascent from the sea at Nieuport to the city of Ipres.

	Feet.
Difference of height which the river from Nieuport to Ipres has above the canal from Nieuport to Plaschendahl, taken cross the harbour of Nieuport, by means of the tides which come up against the outer gates of the sluices next the sea on each,	3
Declivity of the river Iprelee from Nieuport to Boefinghe,	11
Difference of level of the water above and below the sluice of Boefinghe,	$22\frac{2}{3}$
Declivity in the canal from Boefinghe to Ipres,	$\frac{1}{3}$
Total ascent, all one way, from Nieuport to Ipres,	<u>37</u>

Now $84 - 37 = 47$ feet for the difference of height which the surface of the Lys at Commines has above the surface of the canal at Ipres. I have made use of the above example preferably to any others, as it is very complicated, and because the quantities of declivity which I have put down are not arbitrary; and, moreover, because I had the good fortune to find, some years after I had taken those measures, that I only differed two feet

86. A table of the elevation of countries above the surface of the sea, at each 100 miles of length up the course of the principal rivers in the world, as far as they extend; by computation from the principles laid down in this treatise.

Feet of elevation in 100 miles.	Feet of elevation in 200 miles.	Feet of elevation in 300 miles.	Feet of elevation in 400 miles.	Feet of elevation in 500 miles.	Feet of elevation in 600 miles.	Feet of elevation in 700 miles.	Feet of elevation in 800 miles.	Feet of elevation in 900 miles.	Feet of elevation in 1000 miles.	Names of Rivers, and quantities whereby the length of their course is to be diminished, to have the distance from their mouths in a direct line.
100	210	330	470	630	820	1040	1300	1600	1950	The river Trent, the Meander, and many others of the same kind, which are feldom of great extent; in these one-third of the course may be allowed for its deviations from a right line.
150	310	480	670	880	1120	1400	1770	2160	2620	The river Thames; the Seine and the Somme in France; the Nile and the Niger in Africa; the river St. Laurence, the Oronoko, the river of Amazons, the rivers of Paraguay: in these about one-fourth of the length of course may be allowed for turns and windings in it.
220	450	700	980	1290	1640	2040	2500	3030	3630	The Severn, the Loire and Garonne in France; the Tagus, Guadiana and Guadalquivir in Spain; the Po and the Tyber in Italy; the Meuse, Rhine and Elbe in Germany; the Weixel, Neisser, Bog and Nieper in Poland; the Don and Dwina in Russia; the Amur in Tartary; the Hoang-ho-keou and Yang-tie Kiang-keou in China; the rivers of Cambodia, Ava and Ganges in India; the Euphrates; the Zaïre in Congo; the Mississippi: in these may be allowed about one-fifth of the length of the course for turns and windings in it.
300	650	1050	1520	2070	2720	3500	4440	5570	6920	The Rhône in France; the Ebro and Douro in Spain; the Danube; the Wolga; the Irriich, Oby, Jenetca and Lena in Siberia; the Malmitra in Cilicia; the Tigris; the Indus. The course of these rapid rivers is usually very straight, and there cannot be above one-sixth of the length thereof allowed for deviations from a right line.